Calculus Warm Up # 6-4

Find the arc length of $f(x) = \sqrt[3]{x}$ on $[-8, 8]$

Are there any values of x where $f'(x)$ is not continuous? Check the definition of arc length... what can you do about that?

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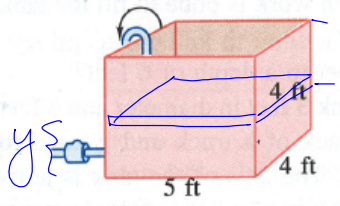
1. Determine the work done in lifting a 100-pound bag of sugar 10 feet.
2. Determine the work done by a hoist in lifting a 2400-pound car 6 feet.
3. A force of 25 pounds is required to slide a cement block on a plank in a construction project. The plank is 12 feet long. Determine the work done in sliding the block along the length of the plank.

11. A rectangular tank with a base 4 feet by 5 feet and a height of 4 feet is filled with water (see figure). (The water weighs 62.4 pounds per cubic foot.) How much work is done in pumping water out over the top edge in order to empty

(a) half of the tank?

(b) all of the tank?



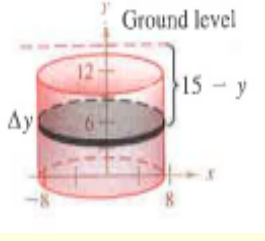
a) $\int_2^4 (1248)(4-y)\,dy$

F

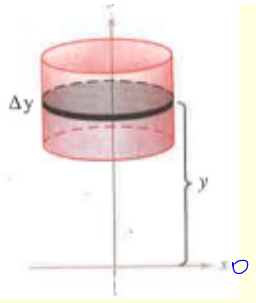
$\Delta F = (62.4)(5 \cdot 4 \cdot \Delta y)$

12. Repeat Exercise 11 for a tank filled with gasoline that weighs 42 pounds per cubic foot.

13. A cylindrical water tank 12 feet high with a radius of 8 feet is buried so that the top of the tank is 3 feet below ground level (see figure). How much work is done in pumping a full tank of water up to ground level?



14. Suppose the tank in Exercise 13 is located on a tower so that the bottom of the tank is 20 feet above the level of a stream (see figure). How much work is done in filling the tank half full of water through a hole in the bottom, using water from the stream?

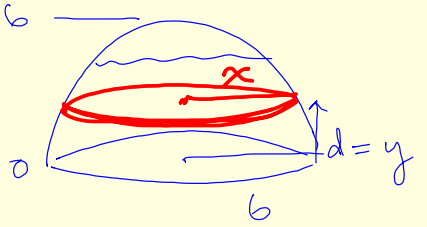


$$W = \int_{20}^{26} (3993.6\pi)\, y \, dy$$

$$W = 551{,}116.8\pi \text{ ft-lbs}$$

$$\Delta F = (62.4)(64\pi\, \Delta y)$$

15. A hemispherical tank of radius 6 feet is positioned so that its base is circular. How much work is required to fill the tank with water through a hole in the base if the water source is at the base?

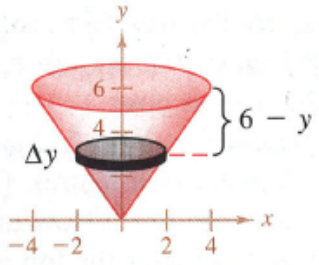


$$\int_0^6 F(x) \cdot d \; dy$$

$$x = \sqrt{36 - y^2}$$

$$\Delta F = (62.4)\left(\pi\left(\sqrt{36-y^2}\right)^2 \Delta y\right)$$

17. An open tank has the shape of a right circular cone (see figure). The tank is 8 feet across the top and is 6 feet high. How much work is done in emptying the tank by pumping the water over the top edge?



6.5

Work done by a constant force

Work done by a variable force

- Work to move liquid
- Work to stretch or compress a spring

Hooke's Law: The force F required to compress or stretch a spring (within its elastic limits) is proportional to the distance d that the spring is compressed or stretched from its original length. That is,

$$F = kd$$

k = spring constant, depends on the nature of the spring... how stiff or thick it is, the type of material it is made out of. It is essentially the force per unit of length needed to change the length of the spring.

You are usually given an initial condition so you can calculate the spring constant first, then find work:

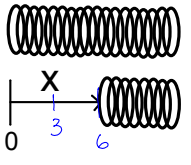
A force of 750 pounds compresses a spring 3 inches from its natural length of 15 inches. Find the work done in compressing the spring an additional 3 inches.

$F = kd$

$750 \text{ lbs} = k(3 \text{ in.})$

$k = 250 \frac{\text{lbs.}}{\text{in.}}$

natural length



$F(x) = kx$

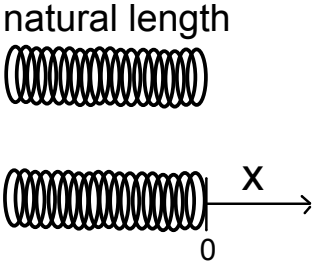
$W = \int_3^6 F(x)\,dx$

$W = \int_3^6 250x\,dx$

$W = \frac{250x^2}{2}\Big]_3^6$

You try: $F = kd$ $W = \int F(x)\,dx$

A 30 pound force is needed to stretch a spring 6 inches from its natural length. Find the work done stretching the spring an additional 4 inches.



$F = kd$

$30 = k(6)$

$k = 5$

$W = \int_6^{10} 5x\,dx$

$W = 160$ in-lbs.

Same spring:

Find the work needed to stretch it 8 in. from its natural length.

$\int_0^8$

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and work on the purple classwork.